SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Find the related rate equation.

1) The kinetic energy $K$ of an object with mass $m$ and velocity $v$ is $K = \frac{1}{2}mv^2$.  

How is $dm/dt$ related to $dv/dt$ if $K$ is constant?

$$0 = \frac{1}{2} \frac{dm}{dt} v^2 + \frac{1}{2} m \frac{dv}{dt} v \frac{dv}{dt}$$

$$\Rightarrow \frac{dm}{dt} = -\frac{2m}{v} \frac{dv}{dt}$$

Solve the problem.

2) A product sells by word of mouth. The company that produces the product has noticed that revenue from sales is given by $R(t) = 5\sqrt{x}$, where $x$ is the number of units produced and sold. If the revenue keeps changing at a rate of $\$400$ per month, how fast is the rate of sales changing when 1600 units have been made and sold? (Round to the nearest whole number.)

$$\frac{dR}{dt} = \frac{5}{2} x^{-\frac{1}{2}} \frac{dx}{dt}$$

$$400 = \frac{5}{2} (1600)^{-\frac{1}{2}} \frac{dx}{dt}$$

$$\Rightarrow \frac{dx}{dt} = 6400 \text{ units per month}$$

3) A piece of land is shaped like a right triangle. Two people start at the right angle at the same time, and walk at the same speed along different legs of the triangle while spraying the land. If the area covered is changing at $3 \text{ m}^2/s$, how fast are the people moving when they are 2 m from the right angle? (Round approximations to two decimal places.)

$$A = \frac{1}{2} x^2$$

$$\frac{dA}{dx} = 2 \frac{1}{2} x \frac{dA}{dx}$$

$$\Rightarrow \frac{dA}{dt} = x \frac{dx}{dt}$$

$$3 = 2 \frac{dx}{dt}$$

$$\Rightarrow \frac{dx}{dt} = \frac{3}{2} = 1.5 \text{ m/sec}$$

4) One airplane is approaching an airport from the north at 193 km/hr. A second airplane approaches from the east at 209 km/hr. Find the rate at which the distance between the planes changes when the southbound plane is 26 km away from the airport and the westbound plane is 15 km from the airport.

$$\frac{dc}{dt} = \frac{dx}{dt} + \frac{dy}{dt}$$

$$c^2 = x^2 + y^2$$

$$c^2 = 26^2 + 15^2$$

$$c = 30.02$$

$$-272 \text{ km/hr} = \frac{dc}{dt}$$
5) The radius of a right circular cylinder is increasing at the rate of 2 in./s, while the height is decreasing at the rate of 9 in./s. At what rate is the volume of the cylinder changing when the radius is 7 in. and the height is 16 in.?

\[
\frac{dr}{dt} = 2 \text{ in./sec} \quad \frac{dh}{dt} = -9 \text{ in./sec} \quad \frac{dV}{dt} = ? \quad r = 7 \text{ inches} \quad h = 16 \text{ inches}
\]

\[
V = \pi r^2 h
\]

\[
\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt} = 2\pi (7)(16)(2) + \pi (7)^2 (-9) = 7\pi \text{ in.}^3/\text{sec}
\]

6) Water is discharged from a pipeline at a velocity \( v \) given by \( v = 1650p^{(1/2)} \), where \( p \) is the pressure (in psi). If the water pressure is changing at a rate of 0.437 psi/second, find the acceleration (\( dv/dt \)) of the water when \( p = 37.0 \) psi.

\[
\frac{dv}{dt} = \frac{1}{2} (1650) \frac{d^2p}{dt^2}
\]

\[
= \frac{1}{2} (1650)(37) \frac{1}{2} (0.437) = 59.27 \text{ ft/ sec}^2
\]

Find the extreme values of the function and where they occur (Please Justify Your Answer Using Calculus)

7) \( y = \frac{4x}{x^2 + 1} \)

\[
y' = \frac{4(x^2+1) - 2x(4x)}{(x^2+1)^2} = -\frac{4(x^2-1)}{(x^2+1)^2}
\]

\[
y' = 0 \quad \text{when} \quad x = \pm 1
\]

(A) Since \( f'(x) \) is going from \( -\) to \( + \), the pt \((-1,-2)\) is a local min (Absolute min)

(B) Since \( f'(x) \) is going from \( + \) to \( - \), the pt \((1,2)\) is the local max (Absolute max)

Find the critical point(s), if any, and determine the local extreme values.
(Please Justify Your Answer Using Calculus)

8) \( y = x^{2/3}(x^2 - 4) \) \( \text{When } x \geq 0 \)

\[
y = \frac{8}{3}x - 4 \text{ } \frac{2}{3}
\]

\[
y' = \frac{8}{3}x - \frac{1}{3} = \frac{8}{3}x - \frac{1}{3} = \frac{8}{3} \frac{x - 1}{x^2/3} = \frac{8}{3} \frac{x - 1}{x^{2/3}} = \frac{8(x^2 - 1)}{3x^{1/3}}\]

\[
y' = 0 \text{ when } x = +1 \text{ or } -1 \quad \text{But } x = -1 \text{ is not in Domain - (x)
}

\[
y' \text{ is undefined when } x = 0
\]

Hence, the c. Numbers are \( x = 0 \) and \( x = 1 \)

Local Minimum \((1, -3)\) (Also Absolute Min)

\( \text{b/c } f'(x) \text{ goes from } - \to + \)