**Math 093 and Math 117A – Linear Functions**

**Life Expectancies at Birth**

1) a) Identify the independent (explanatory) variable and corresponding units.
   b) Identify the dependent (response) variable and corresponding units.
   a) T = x = year of birth in years after 1980 (x = 0 represents the year 1980) = explanatory
   b) L = y = life expectancy of Americans at birth (in years) = response

2) Use the calculator to sketch the scatter-diagram – transfer graph to paper, a fast rough sketch without scale is ok. LABEL axes as in part (1)

3) Use the calculator to find the equation of the linear model that best fit the data; round to 3 decimal places.
   a. Write the calculator equation.
      \[ Y = 0.165 \times + 73.705 \]
   b. Write the equation using L and t and function notation.
      \[ L(t) = 0.165t + 73.705 \]
   c. Give the value of the linear correlation coefficient, r.
      \[ R = 0.994, \text{ positive association} \]
   d. Would you call this association strong or weak? Explain.
      The correlation coefficient is close to 1 which indicates a very strong linear association; the mathematical model can be used for predictions.

4) What is the slope of the linear model? Include units. Interpret within context.
   \[ m = 0.165 \text{ years / years after 1980} \]
   Since 1980, the life expectancy of Americans has been increasing by 0.165 years for every year increase in the birth-year

5) What is the y-intercept of the linear model? Include units. Interpret within context.
   \[ B = 73.705 \text{ years. This is the point (0, 73.7). Since 0 corresponds to the year 1980 we can say the following:} \]
   The model predicts a life expectancy of 73.7 years for Americans born in 1980.
6) What is the life expectancy at birth of an American born in the year 2005? Comment on interpolation/extrapolation. Show all work and answer using a complete sentence within context.

Since 2005 – 1980 = 25,
If x = 25, what is y?
Y = 0.165 (25) + 73.705 = 77.83 years
According to the model, an American born in the year 2005 has a life expectancy of 77.8 years.
This is an example of interpolation because 2005 is within the interval [1980, 2006]

7) Find the residual.

Notice that according to the data, an American born in the year 2005 has a life expectancy of 77.9 years.
Residual = observed – predicted = 77.9 – 77.83 = 0.07
The residual is positive because the observed value is larger than the predicted. This very small residual tells us that the 77.83 is a good prediction.

8) Use the table feature of your calculator to complete the “predicted” values. Then, find the residuals.

<table>
<thead>
<tr>
<th>Year of Birth</th>
<th>t Years after 1980</th>
<th>Life Expectancy (years)</th>
<th>Life Expectancy Predicted (yrs)</th>
<th>Residual (yrs) Observed – Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>0</td>
<td>73.7</td>
<td>73.71</td>
<td>73.7 - 73.71 = -0.01</td>
</tr>
<tr>
<td>1985</td>
<td>5</td>
<td>74.7</td>
<td>74.53</td>
<td>74.7 - 74.53 = 0.17</td>
</tr>
<tr>
<td>1990</td>
<td>10</td>
<td>75.4</td>
<td>75.36</td>
<td>75.4 - 75.36 = 0.04</td>
</tr>
<tr>
<td>1995</td>
<td>15</td>
<td>75.8</td>
<td>76.18</td>
<td>75.8 - 76.18 = -0.38</td>
</tr>
<tr>
<td>2000</td>
<td>20</td>
<td>77</td>
<td>77.01</td>
<td>77 - 77.01 = 0.01</td>
</tr>
<tr>
<td>2005</td>
<td>25</td>
<td>77.9</td>
<td>77.83</td>
<td>77.9 - 77.83 = 0.07</td>
</tr>
<tr>
<td>2006</td>
<td>26</td>
<td>78.1</td>
<td>78.0</td>
<td>78.1 - 78.0 = 0.1</td>
</tr>
</tbody>
</table>

9) What is the year of birth of an American with a life expectancy at birth of 80 years? Comment on interpolation/extrapolation. Show all work and answer using a complete sentence within context.

If y = 80, what is x?
80 = 0.165 x + 73.705
X = 38.15
38 + 1980 = 2018
The model predicts a life expectancy of 80 years for an American born in the year 2018.
This is extrapolation, too far away from 2006 which is the last point on the data. The life expectancy model may be different by then and probably this is not a good prediction.

10) Find the following L(3). Interpret your answer within context.

This is the same as saying: if x = 3, what is y?
Y = 0.165 (3) + 73.705 = 74.2 years
Since x = 3 means year 1983, we can say the following:
The model predicts a life expectancy of 74.2 years for an American born in the year 1983.

11) Solve L(t) = 78. Interpret your answer within context.

This is the same as saying: if y = 78, what is x?
78 = 0.165 x + 73.705
X = 26.03 years after 1980
Year of birth 2006
The model predicts a life expectancy of 78 years for an American born in the year 2006.
12) Interpret within context the statement: Solve L(3) = 74.2.
   If x = 3, y = 74.2
   X = 3 means 3 years after 1980, which is the year 1983
   Y = 74.2 is the life expectancy
   The model predicts a life expectancy of 74.2 years for an American born in the year 1983